Application of Sommerfeld-Integral Expressions in Dielectric and Magnetic Half-Space Problems

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***Abstract*—Entire set of expressions for components of reflected and transmitted EM fields in a lossy dielectric half-space scenario, excited by an elementary electric-current Hertz dipole, is presented in form of magnetic vector-potential Green's functions. Theoretically, by substituting appropriately the dual magnitudes – magnetic with electric vector-potential, and dielectric permittivity with magnetic permeability, dual equations can be derived for magnetic-current excitation. This claim is verified by means of a commercial general-purpose MoM solver based on surface integral equations.**

Keywords— Sommerfeld integrals, moments method, half-space problem

I. Introduction

The century-old half-space problem [1], [2] has evolved from analysis of low frequency power-line currents above real ground, over radio-frequency antennas to microwave and optical systems in layered media [3]. Therefore, the problem has been addressed many times in the open literature. In the past decades, the researchers have been proposing analytical xand numerical methods in order to achieve efficient and accurate solutions for the half-space problem, by tackling the Sommerfeld integrals, which make the foundation of the half-space electromagnetic (EM) field expressions.

Even though the fundamental theory of the Sommerfeld integrals (SIs) is well established and is discussed in many textbooks (e.g. [4], [5]), in this paper we will revise it, by deriving and presenting the entire set of SIs in a compact, and seldom used form that stems from the original integral expression of the Sommerfeld identity. Such compact form will provide not only a comprehensive theoretical and practical insight into the complexity of the half-space problem, but will also represent the basis of our approach in the numerical examples that follow. Most importantly, the aforementioned theoretical consideration will support the main goal of this paper, which is to illustrate effectively the application of the duality of the SI expressions with respect to the type of the source, i.e. electric and magnetic current distributions. The duality property reduces the total number of SIs required to describe arbitrary half-space problems that consider two homogenous and lossy media.

In order to verify our results, we rely on a general-purpose 3-D full-wave MoM solver, WIPL-D [6]. The proposed verification scenarios consider a lossy ground irradiated by metallic or composite sources placed in air above infinite lossy ground.

II. Derivation of Field Components

In this section we will summarize the derivation of the complete set of expressions for EM field due to an arbitrary elementary Hertzian dipole in the vicinity of plane half-space boundary, keeping the original Sommerfeld-identity form [2], [7].

Let us assume two neighboring half-spaces, with dielectric permittivity and magnetic permeability (ε_1,μ_1) and (ε_2,μ_2), respectively (Fig. 1). For operating frequency *f*, the wave numbers $k_1 = 2\pi f\sqrt{\varepsilon_1\mu_1}$ and $k_2 = 2\pi f\sqrt{\varepsilon_2\mu_2}$ follow. We start from Maxwell equations for electric and magnetic field, expressed in terms of magnetic vector-potential, $\mathbf{A}^{\mathrm{e}}$,

$$\mathbf{E}^{\mathrm{e}} = -\mathrm{j}\omega\left[\mathbf{A}^{\mathrm{e}} + \frac{1}{k^2}\Delta\mathbf{A}^{\mathrm{e}}\right], \quad \mathbf{H}^{\mathrm{e}} = \frac{1}{\mu}\nabla\times\mathbf{A}^{\mathrm{e}}. \tag{1}$$

The incident magnetic vector-potential can be expressed as

$$\mathbf{A}_{\mathrm{i}}^{\mathrm{e}} = \frac{\mu_1}{4\pi} I^{\mathrm{e}}\Delta\mathbf{l}\, g_{\mathrm{i}} \ , \quad g_{\mathrm{i}} = \frac{\mathrm{e}^{-\mathrm{j}k_1R_1}}{R_1} \ , \tag{2}$$

where $I^{\mathrm{e}}\Delta\mathbf{l}$ is the electric current moment, superscript $(\cdot)^{\mathrm{e}}$ denotes fields due to electric-currents, and $R_1 = |\mathbf{r}-\mathbf{r}'| = \sqrt{x^2+y^2+(z-z')^2}$ is the position of the observer with respect to the source.

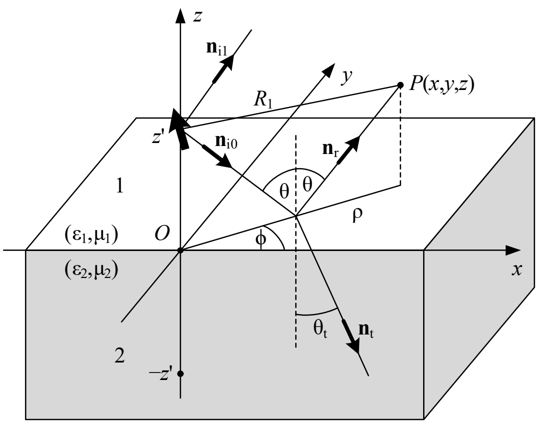

Fig. 1. Scenario with elementary electric or magnetic-current dipole above lossy half-space.

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Let us introduce the unit vector that describes the direction between the source, the half-space interface and the observation point, as shown in Fig. 1,

$$\mathbf{n}_{\mathrm{i}1,0} = \sin\theta\cos\phi\,\mathbf{i}_x + \sin\theta\sin\phi\,\mathbf{i}_y \pm \cos\theta\,\mathbf{i}_z\,, \text{ for } z \begin{smallmatrix}\geq\\ \leq\end{smallmatrix} z'. \qquad (3)$$

We can describe the incident magnetic vector-potential Green's function g_{i} as a sum of *elementary* scalar plane waves using spatial Fourier transform and complex integration [2], [5], [7],

$$g_{\mathrm{i}} = -\mathrm{j}k_1 \int_{\theta=0}^{\pi/2+\mathrm{j}\infty} \int_{\phi=-\pi}^{\pi} \mathrm{e}^{-\mathrm{j}k_1(x\sin\theta\cos\phi + y\sin\theta\sin\phi + |z-z'|\cos\theta)} \sin\theta\mathrm{d}\theta\mathrm{d}\phi\,, \qquad (4)$$

where the exponent is obtained as $\mathbf{R}_1 \cdot \mathbf{n}_{\mathrm{i}}$. By integrating the previous expression over ϕ, we obtain the integral

$$g_{\mathrm{i}} = -\mathrm{j}k_1 \int_{\theta=0}^{\pi/2+\mathrm{j}\infty} J_0(\rho k_\rho)\mathrm{e}^{-\mathrm{j}k_1|z-z'|\cos\theta} \sin\theta\mathrm{d}\theta\,, \qquad (5)$$

where J_0 is the Bessel function of the first kind and zeroth order, $\rho = \sqrt{x^2+y^2}$ is the radial spatial coordinate and k_ρ is the horizontal projection of the wave vector $\mathbf{k}_1$. Using change of variables $k_\rho = k_1\sin\theta$, we obtain the Sommerfeld identity as a real-argument improper integral,

$$g_{\mathrm{i}} = \frac{\mathrm{e}^{-\mathrm{j}k_1R_1}}{R_1} = \int_0^\infty J_0(\rho k_\rho)\mathrm{e}^{-|z-z'|\sqrt{k_\rho^2-k_1^2}} \frac{k_\rho \mathrm{d}k_\rho}{\sqrt{k_\rho^2-k_1^2}}\,. \qquad (6)$$

This basic form will serve for finding the reflected (scattered) and transmitted Green's functions. The totals of magnetic vector-potential in domains 1 and 2 is $\mathbf{A}_1 = \mathbf{A}_{\mathrm{i}} + \mathbf{A}_{\mathrm{r}}$ and $\mathbf{A}_2 = \mathbf{A}_{\mathrm{t}}$, where subscripts $(\cdot)_{\mathrm{r}}$ and $(\cdot)_{\mathrm{t}}$ denote reflected and transmitted potentials.

Let us first assume an excitation in form of elementary z-oriented vertical electric dipole (VED). We can than describe the incident, reflected and transmitted potential as

$$\begin{aligned} &\mathbf{A}_{\mathrm{i}}^{\mathrm{e}} = \frac{\mu_1}{4\pi} I^{\mathrm{e}}\Delta l\, g_{\mathrm{i}}\,\mathbf{i}_z\,,\\ &\mathbf{A}_{\mathrm{r}}^{\mathrm{e}} = \frac{\mu_1}{4\pi} I^{\mathrm{e}}\Delta l\, g_{\mathrm{r},zz}\,\mathbf{i}_z\,, \quad \mathbf{A}_{\mathrm{t}}^{\mathrm{e}} = \frac{\mu_2}{4\pi} I^{\mathrm{e}}\Delta l\, g_{\mathrm{t},zz}\,\mathbf{i}_z\,, \end{aligned} \qquad (7)$$

where $g_{\mathrm{r},zz}$ and $g_{\mathrm{t},zz}$ are the corresponding reflection and transmission Green's functions. Since the reflection angle is the same as the incident angle θ, the reflected propagation unit vector is $\mathbf{n}_{\mathrm{r}} = \mathbf{n}_{\mathrm{i}1}$. On the other hand, the transmitted propagation unit vector is

$$\mathbf{n}_{\mathrm{t}} = \sin\theta_{\mathrm{t}}\cos\phi\mathbf{i}_x + \sin\theta_{\mathrm{t}}\sin\phi\mathbf{i}_y - \cos\theta_{\mathrm{t}}\mathbf{i}_z\,, \qquad (8)$$

and the refraction angle θ_{t} is such that $k_2\sin\theta_t = k_1\sin\theta$. After few algebraic operations, we can assume the form of the reflected and transmitted potential Green's functions

$$\begin{aligned} g_{\mathrm{r},zz} &= \int_0^\infty R_{zz}(k_\rho)J_0(\rho k_\rho)\mathrm{e}^{-\gamma_{z1}(z+z')}\frac{k_\rho\mathrm{d}k_\rho}{\gamma_{z1}}\,,\\ g_{\mathrm{t},zz} &= \int_0^\infty T_{zz}(k_\rho)J_0(\rho k_\rho)\mathrm{e}^{-\gamma_{z1}z'+\gamma_{z2}z}\frac{k_\rho\mathrm{d}k_\rho}{\gamma_{z1}} \end{aligned} \qquad (9)$$

where $\gamma_{z1,2} = \sqrt{k_\rho^2 - k_{1,2}^2}$, $R_{zz}(k_\rho)$ and $T_{zz}(k_\rho)$ are the generalized reflection and transmission coefficients for the z-component of the potential.

Similarly, if we assume an excitation as elementary x-oriented horizontal electric dipole (HED), we can write

$$\begin{aligned} &\mathbf{A}_{\mathrm{i}}^{\mathrm{e}} = \frac{\mu_1}{4\pi} I^{\mathrm{e}}\Delta l\, g_{\mathrm{i}}\,\mathbf{i}_x\,,\\ &\mathbf{A}_{\mathrm{r}}^{\mathrm{e}} = \frac{\mu_1}{4\pi} I^{\mathrm{e}}\Delta l\,(g_{\mathrm{r},xx}\,\mathbf{i}_x + g_{\mathrm{r},zx}\,\mathbf{i}_z)\,,\\ &\mathbf{A}_{\mathrm{t}}^{\mathrm{e}} = \frac{\mu_2}{4\pi} I^{\mathrm{e}}\Delta l\,(g_{\mathrm{t},xx}\,\mathbf{i}_x + g_{\mathrm{t},zx}\,\mathbf{i}_z) \end{aligned} \qquad (10)$$

We note that also y-oriented excitation dipole could have been chosen – this does not reduce the generality of the derivation. The assumed forms of the reflected potential xx- and zx-components should be

$$\begin{aligned} g_{\mathrm{r},xx} &= \int_0^\infty R_{xx}(k_\rho)J_0(\rho k_\rho)\mathrm{e}^{-\gamma_{z1}(z+z')}\frac{k_\rho\mathrm{d}k_\rho}{\gamma_{z1}}\,,\\ g_{\mathrm{t},xx} &= \int_0^\infty T_{xx}(k_\rho)J_0(\rho k_\rho)\mathrm{e}^{-\gamma_{z1}z'+\gamma_{z2}z}\frac{k_\rho\mathrm{d}k_\rho}{\gamma_{z1}}\,,\\ g_{\mathrm{r},zx} &= \cos\phi\,\bar{g}_{\mathrm{r},zx} = \cos\phi\int_0^\infty R_{zx}(k_\rho)J_0(\rho k_\rho)\mathrm{e}^{-\gamma_{z1}(z+z')}\frac{k_\rho^2\mathrm{d}k_\rho}{\gamma_{z1}}\,,\\ g_{\mathrm{t},zx} &= \cos\phi\,\bar{g}_{\mathrm{t},zx} = \cos\phi\int_0^\infty T_{zx}(k_\rho)J_0(\rho k_\rho)\mathrm{e}^{-\gamma_{z1}z'+\gamma_{z2}z}\frac{k_\rho^2\mathrm{d}k_\rho}{\gamma_{z1}}\,, \end{aligned} \qquad (11)$$

where R_{xx}, T_{xx}, R_{zx}, and T_{zx} are the corresponding reflection and transmission coefficients, and $\cos\phi = x/\rho$. If we apply the boundary conditions, $E_{\tan 1} = E_{\tan 2}$ and $H_{\tan 1} = H_{\tan 2}$, to (9) and (11), maintaining the corresponding coordinate systems, we obtain six integral equations which can be solved for the desired reflection and transmission coefficients, resulting in the following set,

$$\begin{aligned} &R_{zz} = \frac{\mu_1 k_2^2\gamma_{z1} - \mu_2 k_1^2\gamma_{z2}}{\mu_1 k_2^2\gamma_{z1} + \mu_2 k_1^2\gamma_{z2}}\,, \quad T_{zz} = \frac{2\mu_1 k_2^2\gamma_{z1}}{\mu_1 k_2^2\gamma_{z1} + \mu_2 k_1^2\gamma_{z2}}\,,\\ &R_{xx} = \frac{\mu_1\gamma_{z1} - \mu_2\gamma_{z2}}{\mu_1\gamma_{z1} + \mu_2\gamma_{z2}}\,, \quad T_{xx} = \frac{2\mu_1\gamma_{z1}}{\mu_1\gamma_{z1} + \mu_2\gamma_{z2}}\,,\\ &R_{zx} = \frac{\mu_2(k_1^2 - k_2^2)}{\mu_1 k_2^2\gamma_{z1} + \mu_2 k_1^2\gamma_{z2}}T_{xx}\,, \quad T_{zx}(k_\rho) = R_{zx}(k_\rho)\,. \end{aligned} \qquad (12)$$

The reflected and transmitted EM field for a VED excitation placed above half-space is given by

$$\mathbf{E}^{\mathrm{e}} = -\frac{\mathrm{j}\omega\mu}{4\pi} I_z^{\mathrm{e}} \Delta l \left\{ \frac{1}{k^2} \frac{\partial^2 g_{zz}}{\partial z \partial \rho} \mathbf{i}_\rho + \left[g_{zz} + \frac{1}{k^2} \frac{\partial^2 g_{zz}}{\partial z^2} \right] \mathbf{i}_z \right\}, \tag{13}$$

$$\mathbf{H}^{\mathrm{e}} = \frac{1}{4\pi} I_z^{\mathrm{e}} \Delta l \left[-\frac{\partial g_{zz}}{\partial \rho} \right] \mathbf{i}_\phi , \tag{14}$$

and for the HED, by

$$\begin{aligned} \mathbf{E}^{\mathrm{e}} = -\frac{\mathrm{j}\omega\mu}{4\pi} I_x^{\mathrm{e}} \Delta l \Bigg\{ & \cos\phi \left[g_{xx} + \frac{1}{k^2} \left(\frac{\partial^2 g_{xx}}{\partial \rho^2} + \frac{\partial^2 \overline{g}_{zx}}{\partial \rho \partial z} \right) \right] \mathbf{i}_\rho \\ & - \sin\phi \left[g_{xx} + \frac{1}{k^2 \rho} \left(\frac{\partial g_{xx}}{\partial \rho} + \frac{\partial \overline{g}_{zx}}{\partial z} \right) \right] \mathbf{i}_\phi \\ & + \cos\phi \left[\overline{g}_{zx} + \frac{1}{k^2} \left(\frac{\partial^2 g_{xx}}{\partial \rho \partial z} + \frac{\partial^2 \overline{g}_{zx}}{\partial z^2} \right) \right] \mathbf{i}_z \Bigg\}, \end{aligned} \tag{15}$$

$$\begin{aligned} \mathbf{H}^{\mathrm{e}} = \frac{I_x^{\mathrm{e}} \Delta l}{4\pi} \Bigg\{ & \sin\phi \left[\frac{\partial g_{xx}}{\partial z} - \frac{1}{\rho} \overline{g}_{zx} \right] \mathbf{i}_\rho \\ & + \cos\phi \left[\frac{\partial g_{xx}}{\partial z} - \frac{\partial \overline{g}_{zx}}{\partial \rho} \right] \mathbf{i}_\phi - \sin\phi_x \left[\frac{\partial g_{xx}}{\partial \rho} \right] \mathbf{i}_z \Bigg\}. \end{aligned} \tag{16}$$

In (13)−(16), for reflected and transmitted field we use the corresponding g_{r} and g_{t} potentials, μ_1 and μ_2, and k_1 and k_2 respectively, according to (9) and (11).

The magnetic vector-potential Green's functions and their derivatives are linear combinations of Sommerfeld integrals (SI), which can be written in a general form,

$$\begin{aligned} & S_{uv}^{l,m,n}(\rho, z, z') = \\ & \int_0^\infty K(k_\rho) J_l(\rho k_\rho) \mathrm{e}^{-\gamma_{z1}|z'| - \gamma_z |z|)} \cdot \left(-\gamma_z \operatorname{sgn} z\right)^m \frac{k_\rho^n \mathrm{d}k_\rho}{\gamma_{z1}}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} & K(k_\rho) = R_{uv}(k_\rho),\ \gamma_z = \gamma_{z1}, \quad \text{for reflected field,} \\ & K(k_\rho) = T_{uv}(k_\rho),\ \gamma_z = \gamma_{z2}, \quad \text{for transmitted field,} \end{aligned} \tag{18}$$

$uv \in \{zz, xx, zx\}$, $l \in \{0,1\}$, $m \in \{0,1,2\}$, and $n \in \{1,2,3\}$. The potential functions and their derivatives appearing in (13)−(16) can be expressed as in Table I, using (17).

Obviously, the VED and HED expressions allow for analysis of arbitrarily oriented elementary Hertzian dipoles, i.e. of arbitrary electric-current distributions. Since the goal of this work is also to model the cases with dielectric objects in the vicinity of the half-space interface, formulas that include magnetic-current sources are required [8].

TABLE I. LIST OF SOMMERFELD INTEGRALS AND GREEN'S FUNCTIONS

VED	
$g_{zz} = S_{zz}^{0,0,1}$	$\frac{\partial g_{zz}}{\partial \rho} = -S_{zz}^{1,0,2}$
$\frac{\partial^2 g_{zz}}{\partial \rho \partial z} = -S_{zz}^{1,1,2}$	$\frac{\partial^2 g_{zz}}{\partial z^2} = S_{zz}^{0,2,1}$
HED	
$g_{xx} = S_{xx}^{0,0,1}$	$\overline{g}_{zx} = S_{zx}^{1,0,2}$
$\frac{\partial g_{xx}}{\partial \rho} = -S_{xx}^{1,0,2}$	$\frac{\partial \overline{g}_{zx}}{\partial \rho} = S_{zx}^{0,0,3} - \frac{1}{\rho} S_{zx}^{1,0,2}$
$\frac{\partial^2 g_{xx}}{\partial \rho^2} = \frac{1}{\rho} S_{xx}^{1,0,2} - S_{xx}^{0,0,3}$	$\frac{\partial^2 \overline{g}_{zx}}{\partial \rho \partial z} = S_{zx}^{0,1,3} - \frac{1}{\rho} S_{zx}^{1,1,2}$
$\frac{\partial^2 g_{xx}}{\partial \rho \partial z} = -S_{xx}^{1,1,2}$	$\frac{\partial \overline{g}_{zx}}{\partial z} = S_{zx}^{1,1,2}$
$\frac{\partial g_{xx}}{\partial z} = S_{xx}^{0,1,1}$	$\frac{\partial^2 \overline{g}_{zx}}{\partial z^2} = S_{zx}^{1,2,2}$

As mentioned before, the problem is dual, which allows us to introduce fields due to magnetic-current Hertzian dipoles,

$$\mathbf{E}^{\mathrm{m}} = -\frac{1}{\varepsilon} \nabla \times \mathbf{A}^{\mathrm{m}} , \qquad \mathbf{H}^{\mathrm{m}} = -\mathrm{j}\omega \left[\mathbf{A}^{\mathrm{m}} + \frac{1}{k^2} \Delta \mathbf{A}^{\mathrm{m}} \right], \tag{19}$$

where $\mathbf{A}^{\mathrm{m}}$ is the electric vector-potential,

$$\mathbf{A}^{\mathrm{m}} = \frac{\varepsilon}{4\pi} I^{\mathrm{m}} \Delta l \, \mathbf{g} , \tag{20}$$

and $\mathbf{g}$ is the corresponding vector Green's function in medium 1 or medium 2, the components of which can assume form similar to (7), (10), and (11). Noting also the formal similarity of (19) and (1), it follows that the field expressions can be derived from (13)−(16) using identites

$$\mathbf{E}^{\mathrm{m}} = -\frac{I^{\mathrm{m}}}{I^{\mathrm{e}}} \mathbf{H}^{\mathrm{e}} , \qquad \mathbf{H}^{\mathrm{m}} = \frac{\varepsilon I^{\mathrm{m}}}{\mu I^{\mathrm{e}}} \mathbf{E}^{\mathrm{e}} , \tag{21}$$

where superscript $(\cdot)^{\mathrm{m}}$ denotes fields due to magnetic-current dipole and I^{m} is magnetic current intensity. Additionally, substitutions $(\mu_1, \mu_2) \to (\varepsilon_1, \varepsilon_2)$ in reflection and transmission coefficients are necessary to completely define the dual case.

III. NUMERICAL EXAMPLES

In order to verify the presented theory, we will consider two simple numerical examples. The computation will be performed within a commercial 3-D EM tool, WIPL-D [6]. In each example, we will compare two simulations: one computed with the MoM and the half-space Green's functions (HSGFs), and the other, computed using the full 3-D MoM approach. The former simulation combines source in free-space with the reflected EM field, derived from the reflected potentials. The calculation of the SIs for lossy half-space is implemented according to [9]. In the latter, reference MoM simulation, the same source is in the presence of an approximated half-space, i.e. a truncated ground clump, realized as an open surface-of-rotation.

A. Dielectrically Coated Dipole Antenna

Let us first consider a scenario with a dielectrically loaded dipole above real ground, i.e. lossy half-space. The reference model with a truncated ground clump is shown in Fig. 2. The dipole is realized as a wire model with radius $r_w = 3$ mm, and arm length $L_{arm} = 0.1$ m, placed at the height $h = 0.6$ m above origin. The dielectric cover is a cylinder of radius $r_d = 15$ mm, height $H_d = 60$ mm, and permittivity of 2. The ground is a lossy dielectric with relative permittivity $\varepsilon_r = 10 - j5$ and permeability $\mu_r = 1$. The finite-sized clump is a disc of radius $A_{disc} = 0.75$ m with a curved rim of radius $r_{rim} = 0.15$ m. The operating frequency band is set from 0.5 to 1 GHz.

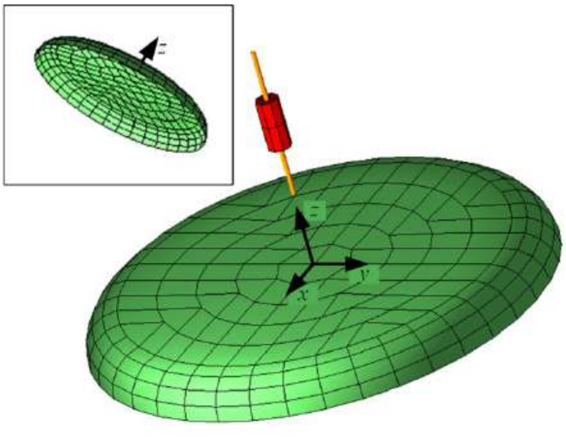

Fig. 2. Reference model of a composite source – a dipole antenna with a dielectric coating (red), above lossy dielectric ground clump (green). The lower half-space is modeled as open dielectric surface (the dipole's size and height are exaggerated for clarity). The inset depicts the bottom side of the clump.

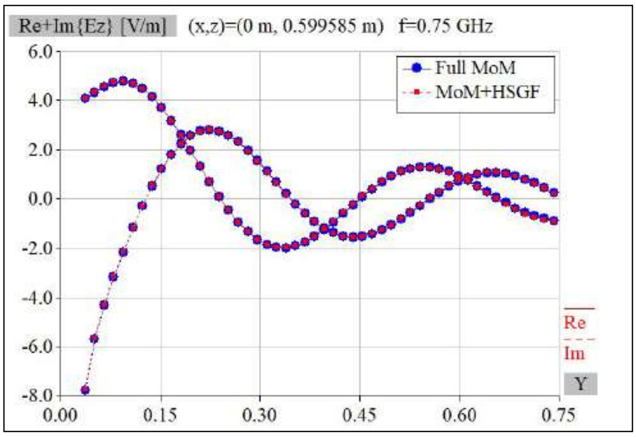

Fig. 3. Real and imaginary part of the electric near field of the coated dipole computed with full MoM (blue) and combined MoM and HSGFs (red) at $f = 0.75$ GHz. The dominant z-component of the field is depicted along the y-axis, for $x = 0$ and $z = h = 0.6$ m.

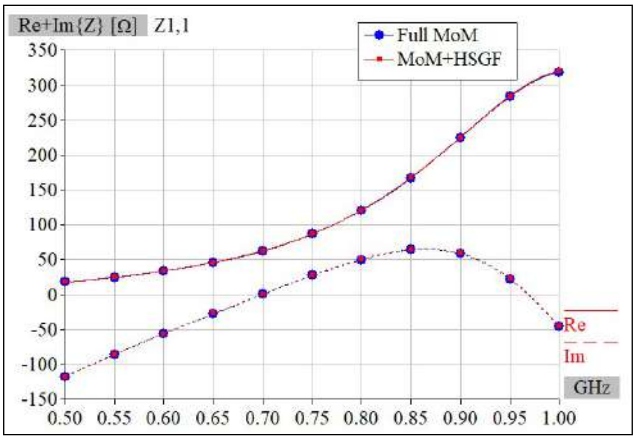

Fig. 4. Real and imaginary part of the impedance of the coated dipole computed with full MoM (blue) and combined MoM and HSGFs (red).

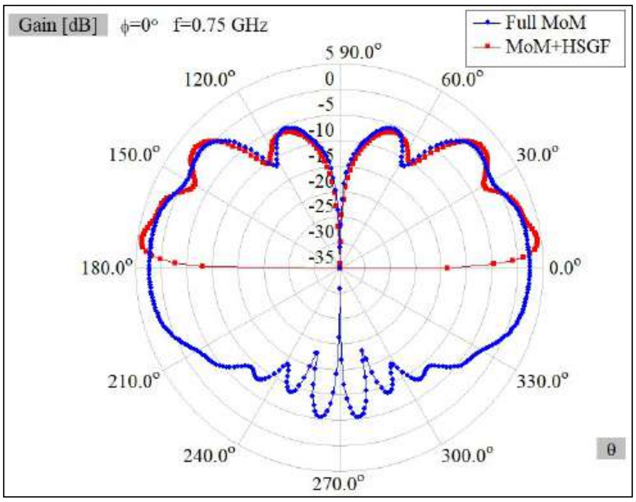

Fig. 5. Gain pattern of the coated dipole computed with full MoM (blue) and combined MoM and HSGFs (red) vs. elevation at f = 0.75 GHz. The pattern is shown in a vertical cut for $\phi = 0$ (the WIPL-D elevation angle of 90° corresponds to positive z-axis).

The electric near field of the dominant z-component and the impendence are shown in Fig. 3 and Fig. 4, respectively. The far field radiation pattern is observed as gain in dBi (Fig. 5). Obviously, the impedance and the near-field correspond very well with the solution obtained using the HSGFs, whereas the far-field diverges slightly more, due to the diffraction effects of the finite ground clump. The effect is particularly pronounced for low elevation angles.

B. Elementary Hertzian Magnetic Dipole

In order to highlight the duality property, an elementary horizontal magnetic dipole (HMD) is observed above the same dielectric half-space as in the previous example. The HMD is realized as an electrically small square loop that approximates a Hertzian unit source, i.e. an infinitesimal magnetic-current moment in x-direction, as shown in Fig. 6. The operating frequency is $f = 1$ GHz, the side length of the loop is $l_m = 1$ mm, and the wire radius is $r_w = 33.3$ µm. The loop is placed at the height $h = 0.03$ m $\approx \lambda/10$, where λ is the wavelength in free space.

The reference model of the dual case is shown in Fig. 7, where an electrically short horizontal electric dipole (HED) is placed above lossy *magnetic* ground with relative permittivity $\varepsilon_r = 1$ and permeability $\mu_r = 10\text{-}j5$. This particular converse case is chosen for its simplicity, since in the expression for the magnetic field in (21) the relative permittivity in the original scenario and the relative permeability of the dual scenario factor out. This does not reduce the generality of the derived expressions. The length of the dipole arm is $L_{arm} = 1$ mm and the wire radius is $r_w = 33.3$ µm.

Let us observe the x-component of the magnetic field along the y-axis ($x = 0$, $y \in [0, 0.75\text{m}]$), and at the height of the dipole ($z = h = 0.03$ m). In Fig. 8, we compare the results for three scenarios: (i) H_x^{m} obtained via the full MoM solution to the ground-clump problem with HMD source (Fig. 6), (ii) H_x^{m} obtained via the HSGF solution to the equivalent half-space problem, and (iii) H_x^{m} obtained indirectly via the HSGF solution to the dual half-space model with magnetic ground, i.e. via E_x^{e}. Close matching of the full MoM solution and the MoM+HSGF solution is observed, confirming the duality expressions (21).

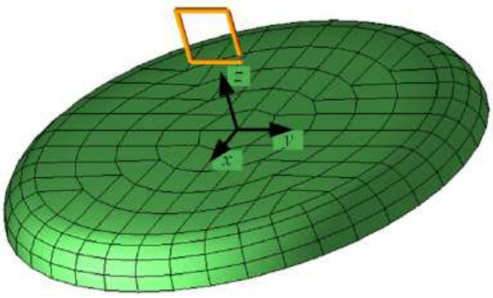

Fig. 6. Reference model for a half-space scenario with an elementary HMD realized as a wire square loop above lossy dielectric ground clump (the dipole's size and height are exaggerated for clarity).

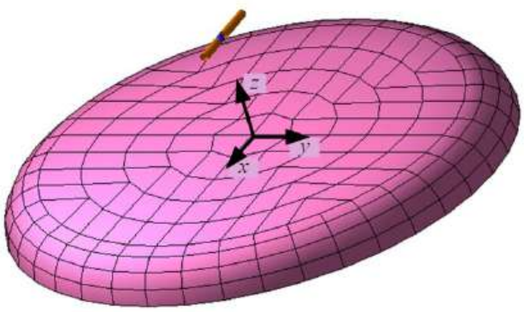

Fig. 7. Reference model for a half-space scenario with an elementary HED realized as a short wire dipole above lossy magnetic ground clump (the dipole's size and height are exaggerated for clarity).

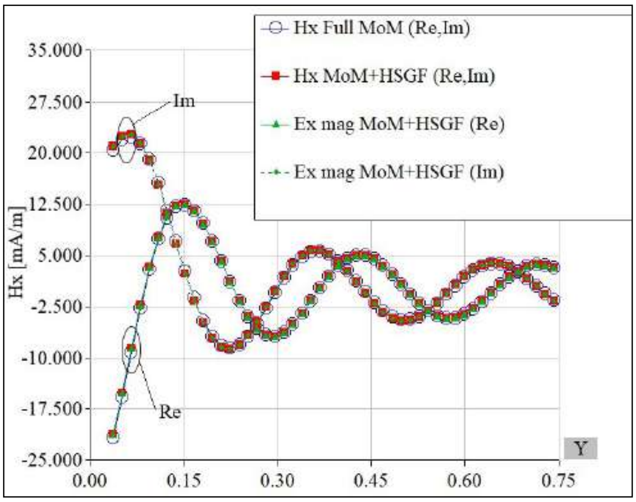

Fig. 8. Real and imaginary part of the magnetic near field of the elementary HMD, computed with full MoM and HSGFs at $f = 1$ GHz. The x-component of the magnetic field is depicted along the y-axis, for $z = h = 33.3$ µm. The green curves designate the real and imaginary part of H_x computed indirectly using the duality expressions (21).

IV. Conclusion

A short, yet comprehensive derivation of the total reflected and transmitted EM field in a lossy half-space problem is presented in terms of magnetic vector-potential Green's functions in cylindrical coordinate system. A general form of the Sommerfeld integrals that constitute the potential Green's functions is introduced, thus allowing for simple notation and computer implementation.

By taking advantage of the duality of the derived field expressions due to electric-current excitation, field expressions due to excitation by magnetic currents are obtained by simple parameter and index substitutions.

The duality of field responses due to electric and magnetic-current sources is successfully verified in numerical examples with both metallic and composite dielectric-metallic radiation sources, using a general-purpose 3-D EM solver.

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